ملاحظة: يمنع استخدام الآلة الحاسبة

مجموع درجات هذا الامتحان 60 درجة

## Question 1:

**Choose the right answer:** **[1 mark each]**

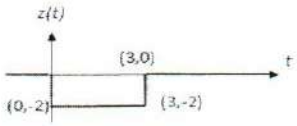
a. أي من الإشارات التالية هي إشارة طاقة:

| | |
|---|---|
| .1 r(t)-r(t-2) | .2 u(t)-u(t-10) |
| .3 sin (3t) | .4 u(t) |

b. $\int_{-1}^{5} e^{-4t^2} cos(t^2)\delta(t-10)dt =$

| | |
|---|---|
| .1 $e^{-400}cos(100)$ | .2 $e^{-400}cos(100)\delta(t-10)$ |
| .3 $e^{-400}sin(100)$ | .4 0 |

c. أي من الصيغ الرياضية الاتية تصف الإشارة z(t) التي بالشكل:




| | |
|---|---|
| .1 -u(t)+u(t-3) | .2 -2u(t).[ u(-t+3)] |
| .3 $-2\text{rect}(\frac{t}{3}-0.5)$ | .4 -2 sinc (3t) |

d. الإشارة x(t)– هي انعكاس الإشارة x(t)حول:

| | |
|---|---|
| .1 المحور الرأسي | .2 المحور الأفقي |
| .3 نقطة الاصل | .4 كل الإجابات خطأ |

e. الزمن الدوري الرئيسي للإشارة $x(t)= 2\cos(t+\pi/4) + 4\sin(\pi t/3)$ هو :

| | |
|---|---|
| .1 $2\pi$ | .2 3 |
| .3 $6\pi$ | .4 كل الإجابات خطأ، الإشارة غير دورية |

f. أي من العبارات الاتية صحيح:

| | |
|---|---|
| .1 كل الأنظمة السببية أنظمة بدون ذاكرة | .2 كل الأنظمة التي بدون ذاكرة أنظمة سببية |
| .3 كل الأنظمة التي بدون ذاكرة أنظمة غير سببية | .4 كل الأنظمة غير السببية أنظمة بدون ذاكرة |

g. أي من الأنظمة الاتية هو نظام سببي:

| | |
|---|---|
| .1 $y(t)= x(-t)$ | .2 $y(t)= x(t)$+t-1 |
| .3 $y(t)= x(t^{1/2})$ | .4 $y(t)= x(t+5)$-5 |



مع تمنياتي للجميع بالتوفيق والنجاح

h. النظام المحدد بالعلاقة $y(t) = x(4 - t)$ هو نظام:

| | |
|---|---|
| 1. متغير مع الزمن | 2. سببي |
| 3. بدون ذاكرة | 4. كل الإجابات خطأ |

i. أي من تحويلات فوريير الآتية صحيح:

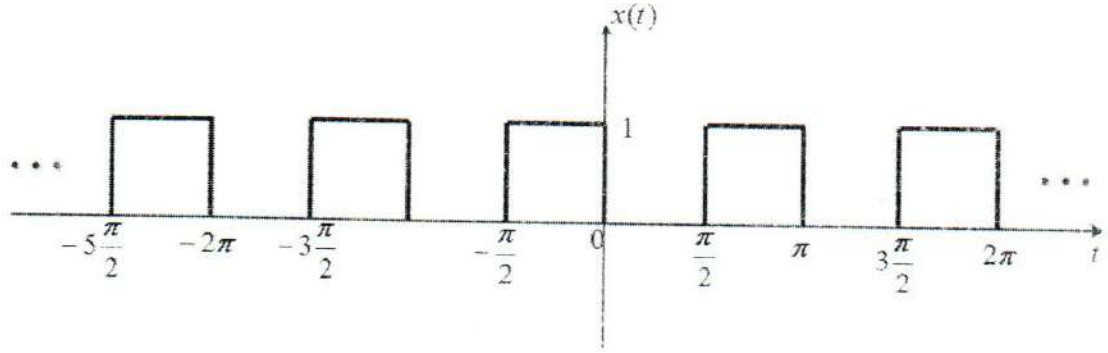
| | |
|---|---|
| 1. $\cos(\pi t) \overset{F.T}{\leftrightarrow} \frac{\pi}{j}[\delta(\omega-\pi)+\delta(\omega+\pi)]$ | 2. $2\delta(t) \overset{F.T}{\leftrightarrow} 2\pi$ |
| 3. $\sin(\pi t) \overset{F.T}{\leftrightarrow} \frac{\pi}{j}[\delta(\omega-\pi)+\delta(\omega+\pi)]$ | 4. $2\delta(t) \overset{F.T}{\leftrightarrow} 2$ |

j. تحوينة فوريير للإشارة $e^{-3t}u(t)$ هي:

| | |
|---|---|
| 1. $X(\omega) = \frac{1}{3-j\omega}$ | 2. $X(\omega) = \frac{1}{3+j\omega}$ |
| 3. $X(\omega) = \frac{3}{9+\omega^2}$ | 4. $X(\omega) = \frac{6}{9+\omega^2}$ |

## Question 2:

i. Compute and sketch the spectrum of the first 5 components of the trigonometric Fourier series for the waveform shown below. [8]




ii. A Fourier series of a periodic signal is given by:

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{1-(-1)^{n+1}}{n\pi}\right]\sin(200\pi nt)$$

a. Find the fundamental frequency in Hertz. [1]
b. Find the average (DC) value of the signal. [1]
c. Sketch the magnitude spectrum of the first (7) coefficients of the signal showing all important values on the sketch. [2]
d. Find the power of the signal using the first (7) coefficients. [2]

**i.** Compute the inverse Laplace Transforms of the following function:

**[4 marks each]**

**a.** $X(s) = \dfrac{4}{(s+1)(s+3)}$ **b.** $F(s) = 9 + \dfrac{s^2+1}{s^2+4}$

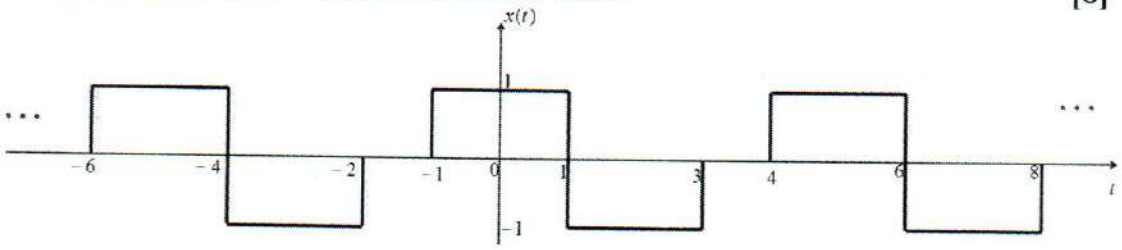
**ii.** Solve the following differential equation using the Laplace transform method

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 4y(t) = 2e^{-t}$$

If $y(0^-) = 2$, *and* $y'(0^-) = 2$ **[8]**

**Question 5:**

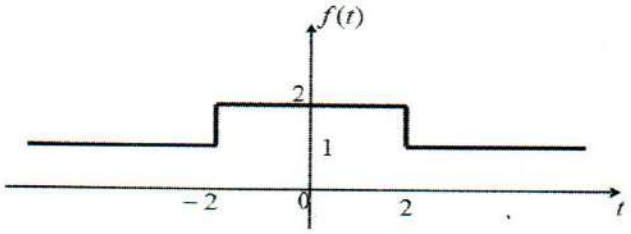
**i.** Compute and sketch the spectrum of the first 5 components of the trigonometric Fourier series for the waveform shown below. **[8]**




**ii.** Using tables and properties, find the Fourier transform and sketch the spectrum of: **[4]**

$$y(t) = [\frac{2}{4+jt}].cos(100t)$$

**iii.** Using tables and properties, find the Fourier transform of the following signal: **[4]**




**iv.** Using tables and **time differentiation property,** Find the Fourier transform of:

$$f(t) = -3[u(t+2) - u(t)] + 3[u(t) - u(t-2)]$$ **[4]**

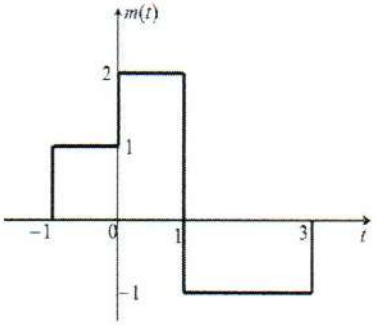
## Question 3:

**i.** Using tables and specified property, Find the Fourier transform of :

**[4 marks each]**

**a.** $x(t) = \frac{8}{4+t^2}$ **(Use duality)**

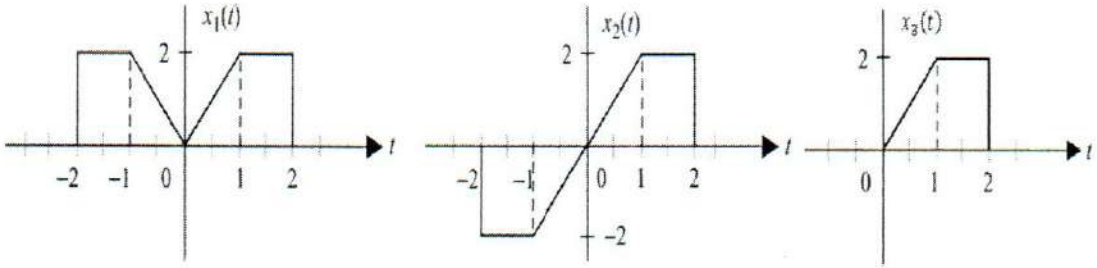
**b.** The shown signal $m(t)$ **(Use time differentiation)**




**ii.** Using tables and properties, find the Fourier transform and sketch the spectrum of:

$s(t) = cos(2t).cos(10t)$ **[4]**

**iii.** For the shown signals $x_1(t), x_2(t)$ *and* $x_3(t)$: **[4]**




If $\boldsymbol{X_1(\omega) = \frac{4}{\omega^2}[\omega \sin(2\omega) + \cos(\omega) - 1]}$

And $\boldsymbol{X_2(\omega) = j\frac{4}{\omega^2}[\omega \cos(2\omega) - \sin(\omega)]}$

Find $\boldsymbol{X_3(\omega)}$

## Question 4:

**i.** Find the Laplace Transforms of the following function: **[4]**

$\boldsymbol{f(t) = [t^3 + 3t^2 + 4t + 3\ ]u(t)}$

## Transformations' table:

| $x(t)$ | $X(\omega)$ | $X(S)$ |
|---|---|---|
| $\delta(t)$ | $1$ | $1$ |
| $1$ | $2\pi\delta(\omega)$ | …… |
| $sgn(t)$ | $\frac{2}{j\omega}$ | …… |
| $u(t)$ | $\pi\delta(\omega)+\frac{1}{j\omega}$ | $\frac{1}{s}$ |
| $e^{-at}u(t)$ | $\frac{1}{j\omega+a}$ | $\frac{1}{s+a}$ |
| $cos(\omega_0 t)$ | $\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$ | $\frac{s}{s^2+\omega_0^2}$ |
| $sin(\omega_0 t)$ | $\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$ | $\frac{\omega_0}{s^2+\omega_0^2}$ |
| $P_t=\begin{cases}A & \lvert t\rvert < a\\ 0 & \lvert t\rvert > a\end{cases}$ Pulse Duration=2a Or $A\,rect(\frac{t}{2a})$ | $2Aa\frac{\sin(\omega a)}{\omega a}$ | …… |
| $te^{-at}u(t)$ | $\frac{1}{(j\omega+a)^2}$ | $\frac{1}{(s+a)^2}$ |
| $t^n e^{-at}u(t)$ | $\frac{n!}{(j\omega+a)^{n+1}}$ | $\frac{n!}{(s+a)^{n+1}}$ |
| $e^{-a\lvert t\rvert}$ | $\frac{2a}{a^2+\omega^2}$ | …… |

## Properties' table:

| Property | Time Domain | Fourier Domain | Laplace Domain |
|---|---|---|---|
| Time shifting | $x(t-t_0)$ | $e^{-j\omega t_0}X(\omega)$ | $e^{-st_0}X(s)$ |
| Frequency Shifting | $e^{j\omega_0 t}x(t)$ or $e^{s_0 t}x(t)$ | $X(\omega-\omega_0)$ | $X(s-s_0)$ |
| Time differentiation | $\frac{dx(t)}{dt}$ | $j\omega X(\omega)$ | $sX(s)-x(0^-)$ |
| | $\frac{d^2x(t)}{dt^2}$ | $(j\omega)^2X(\omega)$ | $s^2X(s)-sx(0^-)-x'(0^-)$ |
| Frequency differentiation | $-tx(t)$ | $-j\frac{dX}{d\omega}$ | $\frac{dX}{ds}$ |
| Duality | $X(t)$ | $2\pi X(-\omega)$ | …… |
| Time convolution | $x_1(t)*x_2(t)$ | $X_1(\omega)X_2(\omega)$ | $X_1(s)X_2(s)$ |

| Trigonometric Fourier Series | Exponential Fourier Series |
|---|---|
| $x(t)=a_0+\sum_{n=1}^{\infty}[a_n\cos(n\omega t)+b_n\sin(n\omega t)]$ | $x(t)=\sum_{n=-\infty}^{\infty}D_n e^{jn\omega t}$ |
| $a_0=\frac{1}{T_0}\int_{T_0}x(t).dt$ | $D_n=\frac{1}{T_0}\int_{T_0}x(t)e^{-jn\omega t}.dt$ |
| $a_n=\frac{2}{T_0}\int_{T_0}x(t)\cos(n\omega t).dt$ | $D_n=\frac{a_n-jb_n}{2}$ |
| $b_n=\frac{2}{T_0}\int_{T_0}x(t)\sin(n\omega t).dt$ | $P_x=\sum_{n=-\infty}^{\infty}\lvert D_n\rvert^2=\lvert D_0\rvert^2+2\sum_{n=1}^{\infty}\lvert D_n\rvert^2$ |