كلية التقنية الالكترونية طرابلس
College of Electronic Technology - Tripoli

القســـم:الاتصالات/التحكم
لطلبة الفصل: السادس

أسئلة الامتحان النهائي لمادة : مواضيع مختارة - DSP
رمز المادة:CT351/ CM308 التاريخ: 29-2-2020

الفصل الدراسي: خريف 2019 اسم الأستاذ: أيمن الترهوني الزمن: ساعة ونصف

Answer all the following questions

Q1. (25 marks) Given $x_1(n) = 1 + 2cos(\pi n) + cos\left(\frac{\pi}{2}n - \frac{\pi}{2}\right)$,

$$x_2(n) = [\ \underline{\mathbf{1}}\ 0\ 1]$$

$$x_3(n) = 2\delta(n) + 2u(n) - 2u(n-1)$$

(i) Find one period of $x_1(n)$.
(ii) Calculate the power of $x_1(n)$.
(iii) Sketch $x_1(n) + x_2(n) + x_3(n)$.
(iv) Find $x_1(n) * x_2(n)$.
(v) Find $X_1(z)\, X_2(z)$.

Hint: the z – transform of $\delta(n) = 1$

Q2. (15 marks) One period of a periodic discrete signal:

$$x(n) = \begin{cases} 1 & for \quad n = 0, 2, 3 \\ 0 & for \quad n = 1 \end{cases}$$

(i) Sketch the discrete signal x(n).
(ii) Use the DFT general formula or the 4-point radix-2 FFT to compute X(k).
(iii) Calculate the power in frequency domain.

Q3. (20 marks) The Pole-Zero plot of a digital filter $H(z)$ is shown in Fig.1.

(i) Make a sketch of the magnitude of the frequency response $|H(\Omega)|$.
(ii) Determine the linear difference equation.
(iii) Draw the implementation structure (Block Diagram) of the Digital filter.

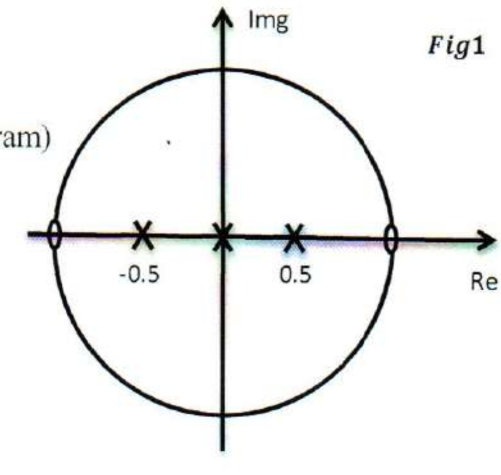

Fig1

تمنياتي للجميع بالتوفيق
أستاذ المادة: أيمن الترهوني

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الاجابة النموذجية للأمتحان النهائي لمادة:
مواضيع مختارة- DSP
الفصل الدراسي: خريف 2019
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Q1. Solution:

(i)

$$\Omega_1 = \pi \rightarrow \quad N = \frac{2\pi K}{\pi} = 2 \;\; for\; K = 1$$

$$\Omega_2 = \frac{\pi}{2} \rightarrow \quad N = \frac{2\pi K}{\frac{\pi}{2}} = 4 \;\; for\; K = 1 \quad , \quad so\;\; N\; total = 4$$

$$x(0) = 1 + 2cos(0) + 2cos\left(-\frac{\pi}{2}\right) = 3 \qquad x(1) = 1 + 2cos(\pi) + 2cos(0) = 0$$

$$x(2) = 1 + 2cos(2\pi) + 2cos\left(\frac{\pi}{2}\right) = 3 \qquad x(3) = 1 + 2cos(3\pi) + 2cos(\pi) = -2$$

$$x_1(n) = [\mathbf{3} \;\; 0 \;\; 3 \;\; -2]$$

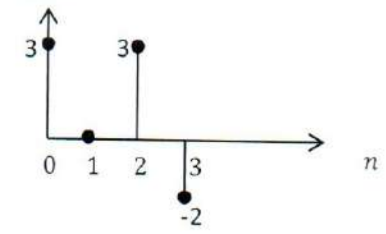
(ii)

$$P_{x\,[n]} = \frac{1}{N}\sum_{n=0}^{N-1} |\, x[n]\,|^2$$

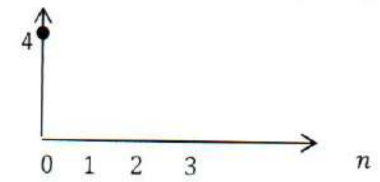
$$= \frac{1}{4}[3^2 + 3^2 + 2^2] = 5.5$$

(iii)

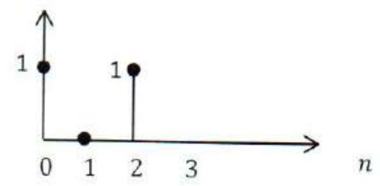
$x_1(n)$



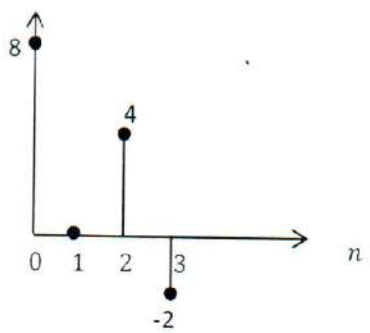
$x_3(n) = 2\delta(n) + 2u(n) - 2u(n-1)$



$x_2(n)$



$x_1(n) + x_2(n) + x_3(n)$



(iv)

	3	0	3	-2
1	3	0	3	-2
0	0	0	0	0
1	3	0	3	-2

$$x_1(n) * x_2(n) = [\,\mathbf{3} \quad 0 \quad 6 \quad -2 \quad 3 \quad 3 \quad -2]$$

(v)

$$X_1(z) = 3 + 3z^{-2} - 2z^{-3}$$
$$X_2(z) = 1 + z^{-2}$$

$$X_1(z)X_2(z) = 3 + 6z^{-2} - 2z^{-3} + 3z^{-4} - 2z^{-5}$$

Q2. Solution:

(i)

(ii)

Method 1: Using DFT General formula

$N = 4\ ,\ \ \Omega = \frac{2\pi}{N} = \frac{\pi}{2}$

$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\Omega nk} \qquad\qquad X[k] = \sum_{n=0}^{3} x[n]e^{-j\frac{\pi}{2}nk}$$

$X[0] = [1 + 0 + 1 + 1] = \mathbf{3}$

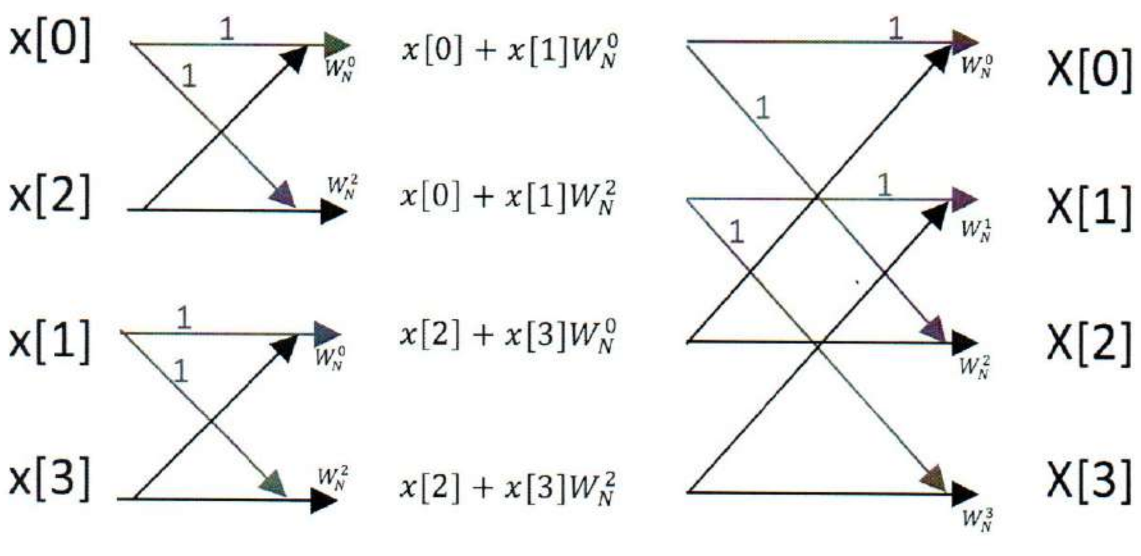
$X[1] = \left[1 + 0e^{-j\frac{\pi}{2}} + 1e^{-j\frac{2\pi}{2}} + 1e^{-j\frac{3\pi}{2}}\right] = j$

$X[2] = \left[1 + 0e^{-j2\frac{\pi}{2}} + 1e^{-j\frac{4\pi}{2}} + 1e^{-j\frac{6\pi}{2}}\right] = 1$

$X[3] = \left[1 + 0e^{-j3\frac{\pi}{2}} + 1e^{-j\frac{6\pi}{2}} + 1e^{-j\frac{9\pi}{2}}\right] = -j$

$$X[k] = [\mathbf{3} \quad j \quad 1 \quad -j]$$

Method 2: Using FFT



N=4

$$W_N^0 = W_4^0 = 1 \ , \qquad W_N^1 = W_4^1 = W_N^{\frac{N}{4}} = -J \ , \qquad W_N^2 = W_4^2 = W_N^{\frac{N}{2}} = -1 \ ,$$

$$W_N^3 = W_4^3 = W_4^1 W_4^2 = W_N^{\frac{N}{4}} W_N^{\frac{N}{2}} = -J(-1) = J$$

$$X[k] = [\mathbf{3} \quad j \quad 1 \quad -j]$$

(iii)

Power $= \frac{1}{N^2}\sum_{K=0}^{N-1}|X(k)|^2$

N=4

$$\boldsymbol{Power} = \frac{\mathbf{1}}{\mathbf{16}}[\mathbf{9+1+1+1}] = \mathbf{0.75}$$

Q3. Solution:

(i)

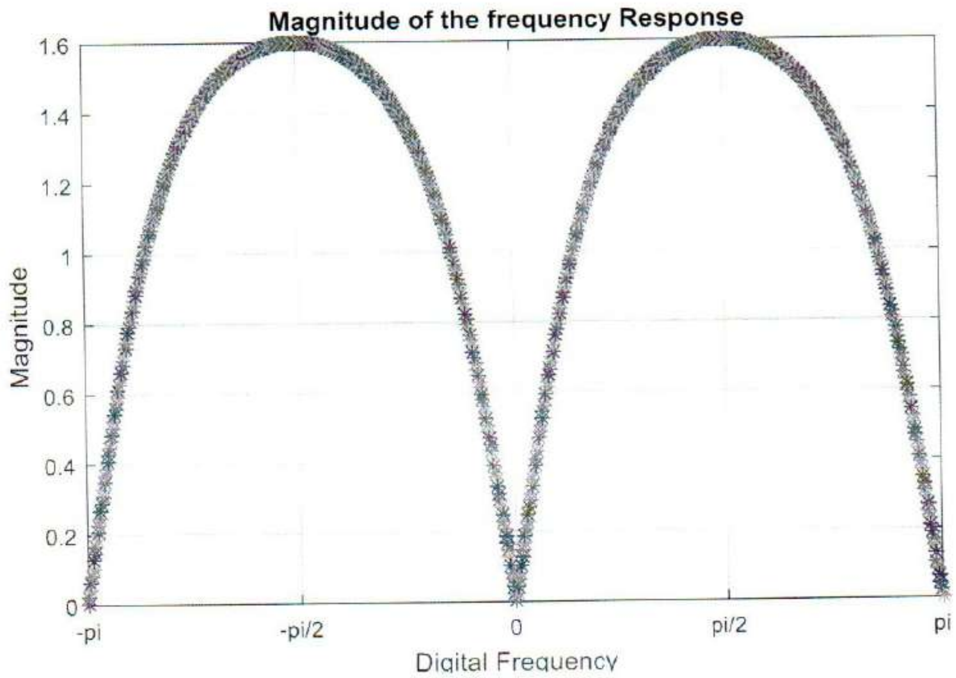
$$H(z) = \frac{(z-1)(z+1)}{z(z-0.5)(z+0.5)} = \frac{z^2-1}{z(z^2-0.25)}$$

$\mathrm{H(z)}|_{z=1} = \mathrm{H}(\Omega)|_{\Omega=0} = 0$

$\mathrm{H(z)}|_{z=j} = \mathrm{H}(\Omega)|_{\Omega=\frac{\pi}{2}} = \frac{j^2-1}{j(\,j^2-0.25)} = \frac{-2}{-1.25j} = \frac{-0.2}{j\ -0.8} \times \frac{J}{J} = j\,1.6$

$|j1.6| = 1.6$

$\mathrm{H(z)}|_{z=-1} = \mathrm{H}(\Omega)|_{\Omega=\pi} = 0$



(ii)

$$\frac{Y(z)}{X(z)} = H(z) = \frac{z^2 - 1}{(z^3 - 0.25z)}\left(\frac{z^{-3}}{z^{-3}}\right) = \frac{z^{-1} - z^{-3}}{(1 - 0.25z^{-2})}$$

$$Y(z) - 0.25Y(z)z^{-2} = X(z)z^{-1} - X(z)z^{-3}$$

$$y(n) - 0.25y(n-2) = x(n-1) - x(n-3)$$

(iii)

$$y(n) - 0.25y(n-2) = x(n-1) - x(n-3) + 0.25y(n-2)$$

One of the possible answers is: